ACT® Coach

Mathematics

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ACT® Lesson Correlation Chart

ACT® Coach, Mathematics Lesson Correlation Chart
according to ACT®'s College Readiness Mathematics Standards, as found here:
http://www.act.org/standard/planact/math/index.html

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Basic Operations and Applications	3, 4, 10, 11
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Numbers: Concepts and Properties	1, 2, 7, 8, 9, 27, 28, 33
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ACT® Coach, Mathematics Lesson Correlation Chart
according to Content Covered by the ACT Mathematics Test, as found here:
http://www.actstudent.org/testprep/descriptions/mathcontent.html

Content Category	*Coach* Lessons
Pre-Algebra	1, 2, 3, 4, 11, 21, 46, 47, 48, 49
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Functions	23, 26, 43, 44, 45

Chapter 1

Number Sense

Compare Real Numbers

The set of **real numbers** consists of the set of **rational numbers** and the set of **irrational numbers**. A rational number is a number that can be written in the form $\frac{a}{b}$, where a and b are **integers** and $b \neq 0$.

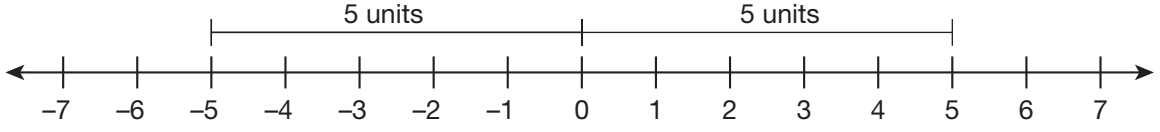
Rational numbers are numbers that can be written as a **ratio** or fraction. All fractions and integers are rational numbers, and so are all terminating or repeating decimals.

Examples of rational numbers:

$-4 = \frac{-4}{1}$ $\qquad$ $2.59 = \frac{259}{100}$ $\qquad$ $0.222... = \frac{2}{9}$

The **absolute value** of a number is its distance from zero on a number line. When you take the absolute value of a number, the result will always be positive, whether the number inside the absolute value symbols is positive or negative.

For example, $|5| = 5$ and $|-5| = 5$ because both 5 and -5 are five units away from zero on a number line.



Example 1

Order the following numbers from least to greatest: 27%, $|-3|$, $-\sqrt{4}$, $\frac{3}{4}$, 1.12.

Strategy **Find the decimal equivalent of each number. Then compare the decimals.**

Step 1 Write each number as a decimal.

$27\% = 0.27$

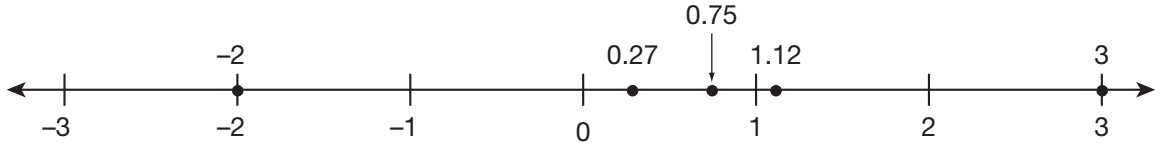
$|-3| = 3$

$-\sqrt{4} = -2$

$\frac{3}{4} = 0.75$

$1.12 = 1.12$

Step 2 Draw a number line, and graph each number on the number line.



Step 3 List the decimals according to their location on the number line.

$-2, 0.27, 0.75, 1.12, 3$

Step 4 Replace the decimals in your list with their original forms.

$-\sqrt{4}, 27\%, \frac{3}{4}, 1.12, |-3|$

Solution **From least to greatest, the numbers are $-\sqrt{4}$, 27%, $\frac{3}{4}$, 1.12, $|-3|$.**

An irrational number is a decimal in which there is no end to the digits (nonterminating) and the digits do not repeat in a predictable pattern (nonrepeating). Irrational numbers cannot be represented as a ratio of two integers.

Examples of irrational numbers:

$\pi = 3.1459265\ldots$ $\sqrt{7} = 2.64575131\ldots$ $1.23456789101112...$

Note that even though there is a pattern to the last number, it is not a repeating pattern. So this number cannot be written as a fraction and is therefore irrational.

Example 2

Are the numbers below rational or irrational? Locate each on a number line.

$2\pi, \frac{7}{4}, \sqrt{10}$

Strategy **Use the definitions of rational and irrational numbers. Then find decimal equivalents or approximations to locate the numbers on a number line.**

Step 1 Determine if the numbers are rational or irrational.

We know that π is an irrational number. An irrational number multiplied by a rational number (other than 0) produces an irrational number. The product will still be nonterminating and nonrepeating. So, 2π is irrational.

All fractions are rational, so $\frac{7}{4}$ is a rational number.

The square root of a number that is not a perfect square is an irrational number.

Since 10 is not a perfect square, $\sqrt{10}$ is irrational.

Step 2 Find decimal equivalents for rational numbers and decimal approximations for irrational numbers.

A common decimal used for π is 3.14. To find the value of 2π, multiply.

$2\pi \approx 2 \cdot 3.14 = 6.28$

To find the decimal equivalent of a fraction, divide.

$\frac{7}{4} = 7 \div 4 = 1.75$

Using your calculator, find $\sqrt{10}$ and round to two decimal places.

$\sqrt{10} \approx 3.16$

Step 3 Draw a number line, and graph each number on the number line.

Use the decimal approximations from Step 2 to graph each number.

Solution **2π and $\sqrt{10}$ are irrational, and $\frac{7}{4}$ is rational. The numbers are shown on a number line in Step 3.**

Coached Example

Find an approximation of $\sqrt{40}$ to one decimal place without using a calculator.

First find the two whole numbers that $\sqrt{40}$ falls between.

The first 8 perfect squares are 1, 4, ____, _____, _____, _____, _____, and _____.

Which two perfect squares does 40 fall between? _____ and _____

Since $\sqrt{36}$ = ___ and $\sqrt{49}$ = ___, $\sqrt{40}$ is between ____ and ____.

Which of the two perfect squares is closer to 40? _____

So, you'll want to try numbers between 6 and 6.5.

Try 6.1: 6.1^2 = _____

Try 6.2: 6.2^2 = _____

Try 6.3: 6.3^2 = _____

Try 6.4: 6.4^2 = _____

The closest approximation is (_____)^2.

So, $\sqrt{40}$ ≈ _____.

Lesson Practice

Choose the correct answer.

1. Which is an irrational number?

A. $\sqrt{81}$

B. $-0.454545...$

C. 10π

D. 3.14

2. Which is correctly ordered from least to greatest?

A. $-\frac{11}{5}, -\pi, -2.98, -\sqrt{7}$

B. $3\frac{1}{4}, \sqrt{16}, \frac{8}{3}, 2\pi$

C. $-\frac{\pi}{4}, -1\frac{1}{2}, -\sqrt{5}, 0$

D. $-\sqrt{10}, -\frac{\pi}{2}, \frac{3}{4}, 0.81$

3. What is the smallest integer greater than $\sqrt{110}$?

A. 7

B. 9

C. 10

D. 11

4. Which number below is the least?

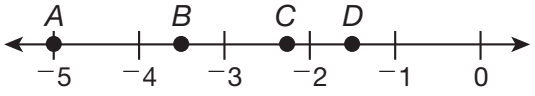
A. $-\sqrt{2}$

B. $-\frac{1}{4}$

C. $|0.04|$

D. 0

Use the number line below for questions 5 and 6.



5. Which point best represents $-\sqrt{5}$?

A. point A

B. point B

C. point C

D. point D

6. Which of the following best describes the location of $-\frac{\pi}{2}$ on the number line?

A. point A

B. point B

C. point C

D. point D

7. Which shows the numbers ordered from least to greatest?

A. $\frac{\pi}{3}, \frac{8}{9}, \sqrt{5}, 1.75$

B. $\frac{8}{9}, \frac{\pi}{3}, 1.75, \sqrt{5}$

C. $\sqrt{5}, \frac{\pi}{3}, \frac{8}{9}, 1.75$

D. $\frac{\pi}{3}, \frac{8}{9}, 1.75, \sqrt{5}$

8. What is the largest integer less than 6π?

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