

ACT[®] Coach

Mathematics



triumphlearning[®]

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ACT[®] Lesson Correlation Chart

ACT[®] Coach, Mathematics Lesson Correlation Chart
 according to ACT[®]'s College Readiness Mathematics Standards, as found here:
<http://www.act.org/standard/planact/math/index.html>

Standard Category	Coach Lessons
Basic Operations and Applications	3, 4, 10, 11
Probability, Statistics, and Data Analysis	46, 47, 48, 49, 50, 51
Numbers: Concepts and Properties	1, 2, 7, 8, 9, 27, 28, 33
Expressions, Equations, and Inequalities	5, 6, 10, 21, 22, 23, 24, 26, 28, 29, 32, 34
Graphical Representations	19, 20, 25, 30, 31, 35, 36, 37
Properties of Plane Figures	12, 14, 15, 18, 38, 39, 40, 41, 42
Measurement	13, 14, 16, 17, 18, 50
Functions	23, 26, 43, 44, 45

ACT[®] Coach, Mathematics Lesson Correlation Chart
 according to Content Covered by the ACT Mathematics Test, as found here:
<http://www.actstudent.org/testprep/descriptions/mathcontent.html>

Content Category	Coach Lessons
Pre-Algebra	1, 2, 3, 4, 11, 21, 46, 47, 48, 49
Elementary Algebra	2, 9, 10, 21, 24, 34, 51
Intermediate Algebra	5, 6, 7, 8, 10, 22, 23, 24, 26, 27, 28, 29, 32, 33, 34
Coordinate Geometry	19, 20, 25, 30, 31, 35, 36, 37
Plane Geometry	12, 13, 14, 15, 16, 17, 18, 38, 39, 40, 41, 42, 50
Trigonometry	43, 44, 45
Measurement	13, 14, 16, 17, 18, 50
Functions	23, 26, 43, 44, 45

Chapter 1

Number Sense

- | | | | |
|-----------------|-----------------------------------|------------------|-------------------------------------|
| Lesson 1 | Compare Real Numbers | Lesson 6 | Solve Problems with Matrices |
| Lesson 2 | Exponents | Lesson 7 | Logarithms |
| Lesson 3 | Compute with Real Numbers | Lesson 8 | Complex Numbers |
| Lesson 4 | Ratios, Proportions, and Percents | Lesson 9 | Use Number Theory to Solve Problems |
| Lesson 5 | Direct and Inverse Variation | Lesson 10 | Math in the Real World |

Compare Real Numbers

The set of **real numbers** consists of the set of **rational numbers** and the set of **irrational numbers**. A rational number is a number that can be written in the form $\frac{a}{b}$, where a and b are **integers** and $b \neq 0$.

Rational numbers are numbers that can be written as a **ratio** or fraction. All fractions and integers are rational numbers, and so are all terminating or repeating decimals.

Examples of rational numbers:

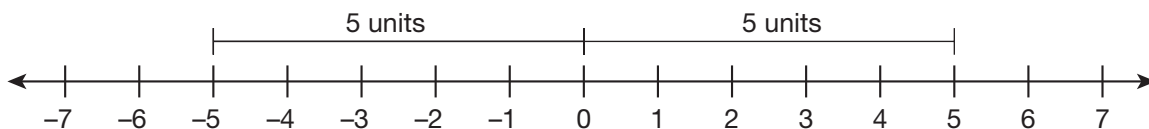
$$-4 = \frac{-4}{1}$$

$$2.59 = \frac{259}{100}$$

$$0.222\dots = \frac{2}{9}$$

The **absolute value** of a number is its distance from zero on a number line. When you take the absolute value of a number, the result will always be positive, whether the number inside the absolute value symbols is positive or negative.

For example, $|5| = 5$ and $|-5| = 5$ because both 5 and -5 are five units away from zero on a number line.



Example 1

Order the following numbers from least to greatest: 27% , $|-3|$, $-\sqrt{4}$, $\frac{3}{4}$, 1.12 .

Strategy Find the decimal equivalent of each number. Then compare the decimals.

Step 1 Write each number as a decimal.

$$27\% = 0.27$$

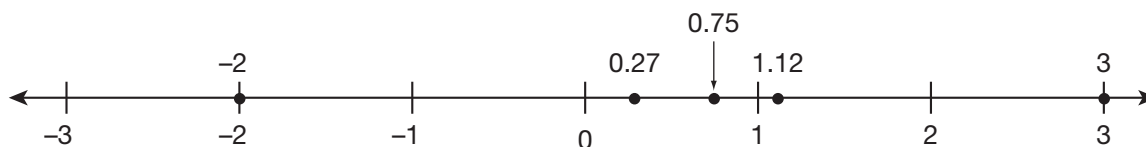
$$|-3| = 3$$

$$-\sqrt{4} = -2$$

$$\frac{3}{4} = 0.75$$

$$1.12 = 1.12$$

Step 2 Draw a number line, and graph each number on the number line.



Step 3 List the decimals according to their location on the number line.

$$-2, 0.27, 0.75, 1.12, 3$$

Step 4 Replace the decimals in your list with their original forms.

$$-\sqrt{4}, 27\%, \frac{3}{4}, 1.12, |-3|$$

Solution From least to greatest, the numbers are $-\sqrt{4}$, 27% , $\frac{3}{4}$, 1.12 , $|-3|$.

An irrational number is a decimal in which there is no end to the digits (nonterminating) and the digits do not repeat in a predictable pattern (nonrepeating). Irrational numbers cannot be represented as a ratio of two integers.

Examples of irrational numbers:

$$\pi = 3.1459265\dots \quad \sqrt{7} = 2.64575131\dots \quad 1.23456789101112\dots$$

Note that even though there is a pattern to the last number, it is not a repeating pattern. So this number cannot be written as a fraction and is therefore irrational.

Example 2

Are the numbers below rational or irrational? Locate each on a number line.

$$2\pi, \frac{7}{4}, \sqrt{10}$$

Strategy Use the definitions of rational and irrational numbers. Then find decimal equivalents or approximations to locate the numbers on a number line.

Step 1

Determine if the numbers are rational or irrational.

We know that π is an irrational number. An irrational number multiplied by a rational number (other than 0) produces an irrational number. The product will still be nonterminating and nonrepeating. So, 2π is irrational.

All fractions are rational, so $\frac{7}{4}$ is a rational number.

The square root of a number that is not a perfect square is an irrational number. Since 10 is not a perfect square, $\sqrt{10}$ is irrational.

Step 2

Find decimal equivalents for rational numbers and decimal approximations for irrational numbers.

A common decimal used for π is 3.14. To find the value of 2π , multiply.

$$2\pi \approx 2 \cdot 3.14 = 6.28$$

To find the decimal equivalent of a fraction, divide.

$$\frac{7}{4} = 7 \div 4 = 1.75$$

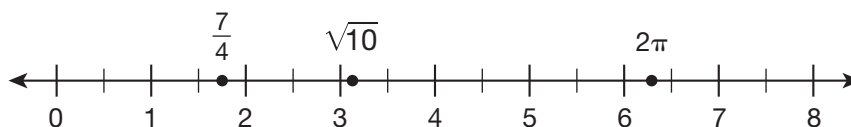
Using your calculator, find $\sqrt{10}$ and round to two decimal places.

$$\sqrt{10} \approx 3.16$$

Step 3

Draw a number line, and graph each number on the number line.

Use the decimal approximations from Step 2 to graph each number.



Solution 2π and $\sqrt{10}$ are irrational, and $\frac{7}{4}$ is rational. The numbers are shown on a number line in Step 3.

Coached Example

Find an approximation of $\sqrt{40}$ to one decimal place without using a calculator.

First find the two whole numbers that $\sqrt{40}$ falls between.

The first 8 perfect squares are 1, 4, _____, _____, _____, _____, _____, and _____.

Which two perfect squares does 40 fall between? _____ and _____

Since $\sqrt{36} = \underline{\quad}$ and $\sqrt{49} = \underline{\quad}$, $\sqrt{40}$ is between _____ and _____.

Which of the two perfect squares is closer to 40? _____

So, you'll want to try numbers between 6 and 6.5.

Try 6.1: $6.1^2 = \underline{\hspace{2cm}}$

Try 6.2: $6.2^2 = \underline{\hspace{2cm}}$

Try 6.3: $6.3^2 = \underline{\hspace{2cm}}$

Try 6.4: $6.4^2 = \underline{\hspace{2cm}}$

The closest approximation is $(\underline{\hspace{1cm}})^2$.

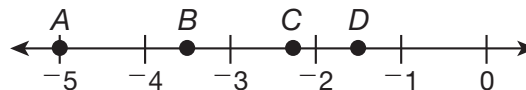
So, $\sqrt{40} \approx \underline{\hspace{1cm}}$.

Lesson Practice

Choose the correct answer.

- Which is an irrational number?
 - $\sqrt{81}$
 - $-0.454545\dots$
 - 10π
 - 3.14
- Which is correctly ordered from least to greatest?
 - $-\frac{11}{5}, -\pi, -2.98, -\sqrt{7}$
 - $3\frac{1}{4}, \sqrt{16}, \frac{8}{3}, 2\pi$
 - $-\frac{\pi}{4}, -1\frac{1}{2}, -\sqrt{5}, 0$
 - $-\sqrt{10}, -\frac{\pi}{2}, \frac{3}{4}, 0.81$
- What is the smallest integer greater than $\sqrt{110}$?
 - 7
 - 9
 - 10
 - 11
- Which number below is the least?
 - $-\sqrt{2}$
 - $-\frac{1}{4}$
 - $|0.04|$
 - 0

Use the number line below for questions 5 and 6.



- Which point best represents $-\sqrt{5}$?
 - point A
 - point B
 - point C
 - point D
- Which of the following best describes the location of $-\frac{\pi}{2}$ on the number line?
 - point A
 - point B
 - point C
 - point D
- Which shows the numbers ordered from least to greatest?
 - $\frac{\pi}{3}, \frac{8}{9}, \sqrt{5}, 1.75$
 - $\frac{8}{9}, \frac{\pi}{3}, 1.75, \sqrt{5}$
 - $\sqrt{5}, \frac{\pi}{3}, \frac{8}{9}, 1.75$
 - $\frac{\pi}{3}, \frac{8}{9}, 1.75, \sqrt{5}$
- What is the largest integer less than 6π ?
