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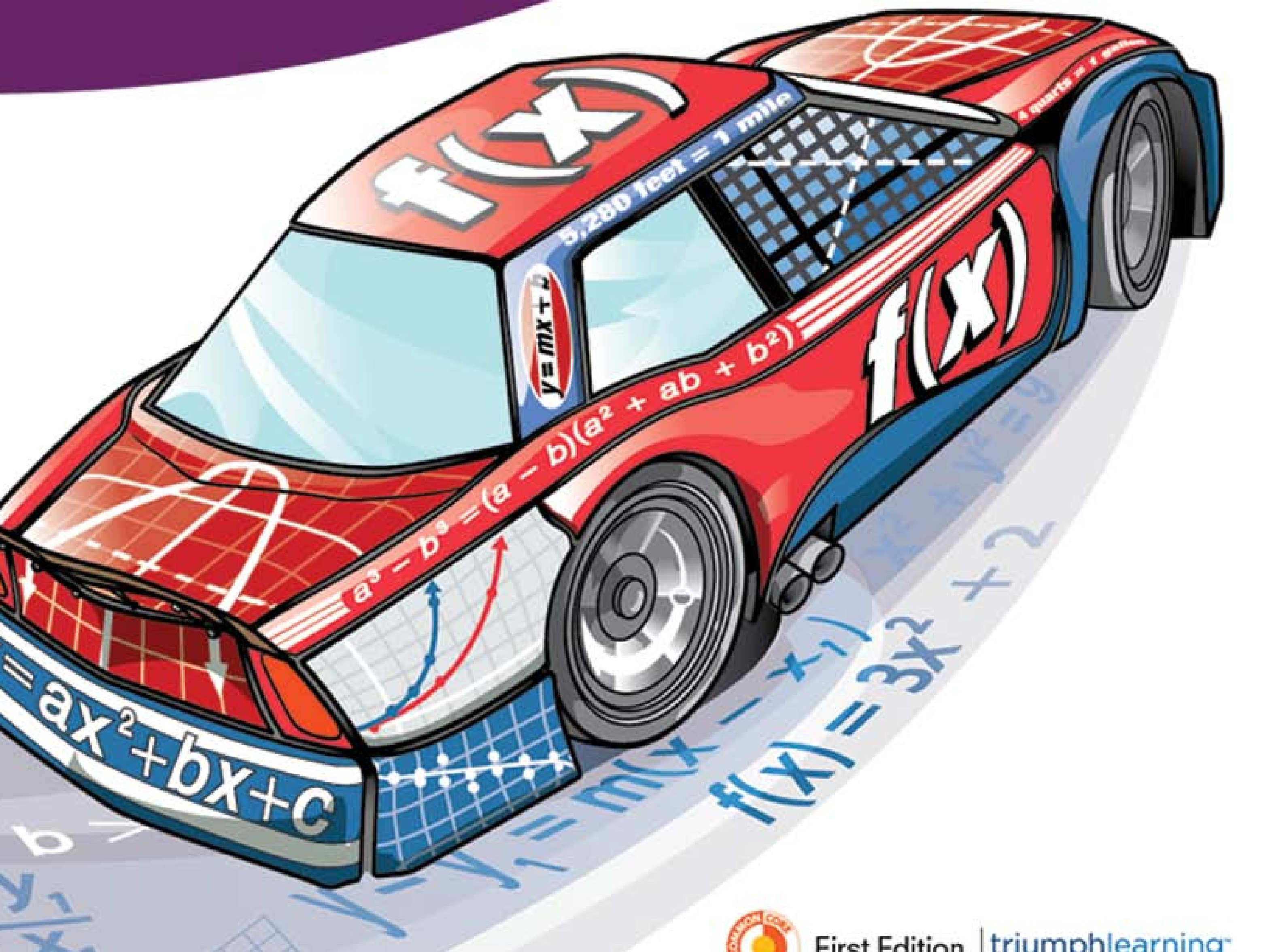
Common Core Coach

Algebra I



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Problem Solving Performance Task

LESSON 1

Units and Dimensional Analysis

When solving a problem, it is important to correctly identify the units being considered or measured. This may require converting a quantity given in one unit to a different unit. To do so, use **conversion factors**, such as 12 inches per foot or 0.001 meter per millimeter, to write a multiplication expression. Be sure to set up the conversion factors correctly so the result is stated in the appropriate units.

Paying attention to the units can help ensure that you perform the conversion correctly. Remember that 1,000 meters = 1 kilometer. Convert 8 meters to kilometers.

Try $\frac{1{,}000\text{ m}}{1\text{ km}}$: $\quad \frac{8\text{ m}}{1} \times \frac{1{,}000\text{ m}}{1\text{ km}} = 8{,}000\,\frac{\text{m}^2}{\text{km}}$

The units in the result, $\frac{\text{m}^2}{\text{km}}$, are not the appropriate units.

Try $\frac{1\text{ km}}{1{,}000\text{ m}}$: $\quad \frac{8\,\cancel{\text{m}}}{1} \times \frac{1\text{ km}}{1{,}000\,\cancel{\text{m}}} = 0.008\text{ km}$

The meter units cancel. The result is in kilometers, the correct unit.

You can convert units within a system of measurement or between different systems of measurement. Though it may require several steps, any unit of measure can be converted to another unit that measures the same property (length, volume, speed, and so on).

Examining the units as you perform calculations is a form of **dimensional analysis**. Dimensional analysis can aid in writing equations by determining how certain quantities can be combined. For example, to add or subtract two quantities, they must be expressed in the same units.

$7\text{ cm} + 1\text{ in.} \neq 8\text{ cm or }8\text{ in.} \qquad 7\text{ cm} + 2.54\text{ cm} = 9.54\text{ cm}$

When multiplying or dividing quantities, units can combine or cancel out. Using dimensional analysis will ensure that you combine quantities by using operations that result in an answer that makes sense. This is especially helpful with rates of change.

Suppose you eat 3 apples per week. How long will it take you to eat a bag of 12 apples? The answer will be in some unit of time. Can you solve by multiplying the quantities?

$$\frac{3\text{ apples}}{1\text{ week}} \times 12\text{ apples} = \frac{36\text{ apples}^2}{\text{week}}$$

This answer above is not given in a unit of time, so try dividing the quantities.

$$\frac{3\text{ apples}}{1\text{ week}} \div 12\text{ apples} = \frac{3\,\cancel{\text{apples}}}{1\text{ week}} \times \frac{1}{12\,\cancel{\text{apples}}} = \frac{0.25}{\text{week}}$$

This is not a unit of time either. Try swapping the terms.

$$12\text{ apples} \div \frac{3\text{ apples}}{1\text{ week}} = 12\,\cancel{\text{apples}} \times \frac{1\text{ week}}{3\,\cancel{\text{apples}}} = 4\text{ weeks}$$

A week is a unit of time, so this calculation makes sense.

Connect

A police officer saw a car travel 1,800 feet in 30 seconds. The speed limit on that road is 55 miles per hour (mph). Was the car speeding?

1 Determine the units given and the units desired.

The car traveled 1,800 feet in 30 seconds, so the speed can be found in feet per second. To compare this rate to the speed limit, we need to convert it to miles per hour.

2 Find the necessary conversion factors.

The distance conversion is from feet to miles. There are 5,280 feet in a mile.

The time conversion is from seconds to hours. There are 60 seconds in a minute and 60 minutes in an hour.

3 Write a dimensional analysis expression.

Remember that the result should be in miles per hour. It may help you to first set up an expression using only the units, so you can see how the units will cancel.

$$\frac{\cancel{\text{ft}}}{\cancel{\text{s}}} \times \frac{\cancel{\text{s}}}{\cancel{\text{min}}} \times \frac{\cancel{\text{min}}}{\text{h}} \times \frac{\text{mi}}{\cancel{\text{ft}}} = \frac{\text{mi}}{\text{h}}$$

Now write the expression with numbers.

$$\frac{1{,}800 \text{ ft}}{30 \text{ s}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{1 \text{ mi}}{5{,}280 \text{ ft}}$$

4 Evaluate the expression. Determine whether the car was driving faster than the speed limit.

$$\frac{1{,}800 \cancel{\text{ ft}}}{30 \cancel{\text{ s}}} \times \frac{60 \cancel{\text{ s}}}{1 \cancel{\text{ min}}} \times \frac{60 \cancel{\text{ min}}}{1 \text{ h}} \times \frac{1 \text{ mi}}{5{,}280 \cancel{\text{ ft}}} \approx 41 \frac{\text{mi}}{\text{h}}$$

▶ The car was traveling at about 41 mph, which is slower than the speed limit of 55 mph. No, it was not speeding.

TRY

A car burns 0.85 gallon of gas per hour when idling. Express this rate in quarts per minute. Round your answer to three decimal places.

Dina took part in a diving competition. She dove 5 times, and her scores were 8.8 points, 9.0 points, 8.6 points, 9.5 points, and 9.2 points. If she calculates her score on an average dive, in what units should the answer be given?

1 Plan the calculation.

To find the average score, add up the individual scores. Then divide by the total number of dives.

2 Analyze the units.

All five scores have the same units: points. Adding them together produces a sum with the same units: points. Dividing that quantity by 5 dives will produce a quantity in points per dive.

▶ The average score should be given in points per dive.

A hospital's records indicate that, on average, 23% of babies born there are delivered by cesarean section. A total of 217 babies were born at the hospital last year, and a total of 220 were born this year.

What should be the expected number of babies born by cesarean section over both years?

1 Identify the quantities in the problem.

The numbers 217 and 220 stand for the numbers of babies born in the given years, so they can be written as 217 babies and 220 babies.

The number 23% expresses the number of babies born by cesarean section out of all of the babies born. Remember that % stands for $\frac{1}{100}$. This quantity can be written as $\frac{23 \text{ cesarean births}}{100 \text{ babies}}$.

The question asks for a number of cesarean births.

2 Write expressions that give the answer in the desired units.

The quantities 217 babies and 220 babies have the same units, so they can be added together to find the number of babies born over both years.

$217 \text{ babies} + 220 \text{ babies} = 437 \text{ babies}$

Now set up a multiplication expression to find the number of cesarean births.

$437 \text{ babies} \times \frac{23 \text{ cesarean births}}{100 \text{ babies}} \approx$

101 cesarean births

▶ About 101 babies were expected to be born by cesarean section over that time.

DISCUSS

Can you think of a situation in which a quantity could be given in pounds of vegetables per day?

Choosing the correct units is important for displaying data in charts and graphs.

The table on the right shows the quarterly profits for a company over a 1-year period.

Make a bar graph to display the data in the table.

Quarter	Profit
1	$167,581
2	$232,191
3	$97,502
4	$124,441

1 Choose a scale for the horizontal axis.

When time is a variable in a given data set, it is generally best to put the data for time on the horizontal axis. In this case, time is given in quarters, labeled 1 through 4. It makes sense to use a scale of 1 quarter.

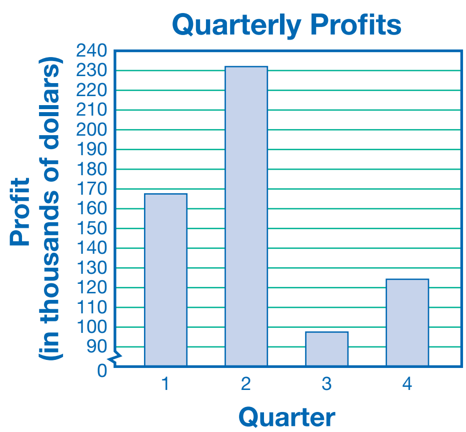
2 Choose a scale for the vertical axis.

The other variable is profit, which is given in dollars. You could make a scale of $1, but that would result in a very tall bar graph.

The least amount is $97,502 and the greatest is $232,191. Given those numbers, it makes sense to use a scale in thousands of dollars and to begin the graph at $90,000. This means that you will be graphing numbers around 168, 232, 98, and 124.

3 Construct the graph.

Remember to label the axes.



DISCUSS

Why might using a scale of hundreds of thousands of dollars make the graph misleading?

Practice

Write an expression for each conversion.

1. There are 1,000 grams in a kilogram. How would you convert 600 kilograms into grams?

2. There are 8 pints in a gallon. How would you convert 13 pints into gallons?

3. A cupcake shop sells an average of 14 dozen cupcakes a day to about 50 customers. What is their average sales rate, in cupcakes per customer?

HINT There are 12 units in a dozen.

4. Write an expression that converts 50 liters per minute into milliliters per second.

REMEMBER The unwanted units should cancel out and only the desired units should remain.

Choose the best answer.

5. The owner of a pool cleaning business wants to know how much time, on average, his workers spend cleaning a pool. Last week, 7 employees each worked a 6-hour shift. In all, they cleaned 42 pools. Which is the most appropriate unit in which to calculate an answer to his question?

A. pools per employee

B. pools per day

C. employees per pool

D. hours per pool

6. Which is equivalent to 21.76 grams per minute?

A. 1.306 kg/h

B. 13.06 kg/h

C. 36.267 kg/h

D. 362.67 kg/h

Solve.

7. While traveling in England, Sonia noticed that the price of gas was 1.4 pounds (£) per liter. She wondered how that compares to the price of gas in Atlanta, where she lives. On that day, the exchange rate was £1 = \$1.56. Set up and evaluate a conversion expression to find the equivalent price in dollars per gallon. Use the conversion factor 1 L = 0.26 gal.

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8. Ravi has started a business importing handwoven and embroidered linen from India. His Indian supplier charges him 460 rupees per meter for the fabric. He wants to make a profit of \$4 per yard. How much must Ravi charge per yard for the imported fabric? Use the following information:
1 dollar = 57.3 rupees
1 meter = 1.09 yards

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Use the table shown below for questions 9 and 10.

The following data show the population in a small town starting with the year 1980.

Year	1980	1985	1990	1995	2000	2005	2010
Population	2,782	3,219	3,788	4,490	5,176	6,490	6,151

9. To graph the data in a line graph, what units would you use for the horizontal axis? How would you label the axis? What scale would you use? ______________________________

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10. To graph the data in a line graph, what units and scale would you use for the vertical axis?

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11. The graph on the right shows the recorded heights of a tomato plant grown in a laboratory.

How can you interpret the origin of the graph?

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