



common core
**Performance
Coach**



Sample Lesson

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Solving Problems with Ratio and Percent

Student Edition pages 31–40

LESSON OVERVIEW

Objectives

Students will:

- Use proportions to solve multi-step percent problems about tax, tips, and discounts
- Solve problems using the simple interest formula
- Solve problems involving percent increase, percent decrease, and percent error

Discussion Questions

- MP1** How can you be sure you have answered the question that was asked in a multi-step problem?
- MP7** What is the difference between percent increase and percent decrease?
- MP3** When is a percent of a number greater than the original quantity? Give an example to illustrate your answer.

Differentiation

Lesson Support Have students use the Math Tool: Percent Rules to make visual models. These models can be used to determine if an answer is reasonable. For instance, in Example 2, students must find 15% of \$53.74. They can mark one of the percent rules to show \$50 as the total. Then each small box represents \$5. Fifteen percent is halfway between 10% and 20%, so the answer should be close to halfway between \$5 and \$10. This technique shows that the computed tip, \$8.06, is reasonable.

Lesson Extension Many problems in this lesson are solved by finding a percent using multiplication and then adding it to the original amount. Show students that an alternate strategy for Example 1 is to add first; that is, add $100\% + 8\%$ to get 108% , and then multiply 1.08 by 45 to find 108% of $\$45$. Challenge students to find other problems in the lesson for which this strategy is appropriate.

Standard

7.RP.3

Key Terms

commission	percent increase
interest	percent of change
percent	principal
percent decrease	simple interest
percent error	

Material

- Math Tool: Percent Rules, p. C5 (Student Edition, p. 311)

1 GETTING THE IDEA

Lesson Opener

A familiar everyday application of percents for students is items on sale. Ask: *What are some different ways that stores advertise items on sale?* (percent off; fractional part off; buy one, get one free)

Have students share stories of great buys they have found on sale items. When possible, have students describe their discounts in terms of percents. For example, ask: *What is the percent off the total regular price when you buy one and get another at the same*

price free? (50%) Then guide students through the Getting the Idea section.

► Example 1

This example shows how a percent of a number can be found and then used to solve a multi-step problem. Relate the strategy in this example back to what students already know about ratios and proportions from previous lessons, and emphasize that percents are ratios of numbers to 100. It may help to relate this idea to students' real-world experiences as well. Ask: *Imagine that a video game has a price tag that reads \$45? Is that what you will actually pay for the game? If you have exactly \$45 in cash, can you buy this game?* (No, if students live in a state with sales tax on luxuries such as games)

▲ Common Errors *When solving multi-step problems, students may begin solving the problem but stop before they are finished. Emphasize that students should always reread the original problem and make sure they have answered the question that was asked.*

► Example 2

Tell students that 15% tips are often computed using mental math, first by finding 10% and then by adding half of that amount. For example, a 15% tip on \$40 equals \$4 + \$2, or \$6. You may wish to have students practice using mental math to compute 15% tips.

▲ Journal Prompt MP1 *Another way to find 15% of \$53.74 is to multiply the dollar amount by the decimal 0.15. How is this equivalent to solving the proportion $\frac{15}{100} = \frac{x}{53.74}$?*

► Example 3

This example differs from previous examples because the amounts are given and students must calculate the percent.

► Examples 4 and 5

These examples require students to compute simple interest and a percent error. Example 4 also shows that another way to find the percent of a number is to multiply the decimal value of the percent—the interest rate—by the amount. If students prefer working with decimals, suggest that they use decimals to write ratios for Example 5 instead of fractions.

▲ ELL Support *The words principal and principle are homophones. Different meanings include:*

principal: *the head of a school; "The principal is in the office."*

principal: *the amount invested or loaned; "The principal invested was \$2,000."*

principle: *a basic truth or law; "I learned an important physics principle."*

2 COACHED EXAMPLE

Monitor students as they work through the Coached Example. As needed, assist students in cross-multiplying to find the value of each percent. Encourage any students who are struggling to draw

loops around the numbers to be cross-multiplied; this method will help them keep track.

For answers, see page A3.

3 LESSON PRACTICE

Before students solve problem 3, make sure students understand the term *commission*. Explain that a commission is a percentage of total sales and is part of a salesperson's earnings. As students are working, pay special attention to problem 6 since it involves

working with non-integer percents. Be sure students can represent decimal percents such as 4.2% and 0.75% as ratios or decimals.

For answers, see page A3.

Solving Problems with Ratio and Percent

1 GETTING THE IDEA

Ratios and proportions can be used to solve many real-world problems, including **percent** problems. A percent is a ratio that compares a number to 100. For example, 20% can be represented as $\frac{20}{100}$.

There are many everyday examples of percent problems. Percents are used to calculate the total cost of a restaurant meal, including sales tax. Sales tax is a percentage of the total cost of the meal. In a sit-down restaurant, a customer also pays a tip, or gratuity, to the server. This is also a percentage of the total amount of the bill.

A salesperson's earnings may be calculated by adding the person's base pay or salary to a **commission**. The commission is a percentage of the total amount of the sales made.

Since percents are ratios, proportions can be used to solve real-world percent problems.

Example 1

The sales tax on a coat that is priced at \$45 is 8%. Find the total cost of the coat.

Strategy Set up and solve a proportion. Then add the amount of tax to the cost of the item.

Step 1 Write the percent as a ratio.

$$8\% = \frac{8}{100}$$

Step 2 Write a ratio comparing the amount of the tax to the price of the coat.

The amount of the tax is unknown. The price of the coat is \$45.

$$\frac{x}{45}$$

Step 3 Set the two ratios equal and solve for x .

$$\frac{8}{100} = \frac{x}{45}$$

$$8 \cdot 45 = 100 \cdot x$$

$$360 = 100x$$

$$3.60 = x$$

So, the amount of the tax is \$3.60.

Step 4

Add the amount of the tax to the cost of the coat.

$$\$45.00 + \$3.60 = \$48.60$$

Solution

The total cost of the coat is **\$48.60**.

Example 2

Nancy and two friends paid a 15% tip for a restaurant bill of \$53.74. Then they split the total evenly. How much did each person pay?

Strategy

Set up and solve a proportion. Then add the amount of the tip to the bill and divide.

Step 1

Set up a proportion to find the amount of the tip.

Write a ratio for the percent: $15\% = \frac{15}{100}$

Write a ratio comparing the unknown amount of the tip to the cost of the bill: $\frac{x}{53.74}$

Set the ratios equal: $\frac{15}{100} = \frac{x}{53.74}$

Step 2

Solve the proportion to find the amount of the tip.

$$\frac{15}{100} = \frac{x}{53.74}$$

$$15 \cdot 53.74 = 100 \cdot x$$

$$806.1 = 100x$$

$$8.061 = x$$

Round the amount. The tip was about \$8.06.

Step 3

Find the amount each person will pay.

Add to find the total of the bill plus the tip.

$$53.74 + 8.06 = 61.80$$

There are 3 people—Nancy plus 2 friends. So, divide the total by 3.

$$61.80 \div 3 = 20.60$$

Solution

Each person will pay **\$20.60**.

A **percent of change** is the ratio of the change in a quantity to the original quantity, expressed as a percent. If the quantity is increasing, this is called a **percent increase**. If the quantity is decreasing, it is a **percent decrease**. A discount or markdown is a type of percent decrease. A markup is a type of percent increase.

Example 3

A lamp that originally cost \$180 was marked down to \$150. Find the percent of the discount.

Strategy Subtract to find the amount of the discount. Set up and solve a proportion to find the percent.

Step 1 Subtract to find the amount of the discount.

$$180 - 150 = 30$$

Step 2 Set up a proportion.

Write a ratio to show the unknown percent. Then write a second ratio comparing the amount of the discount to the original cost.

$$\frac{x}{100} = \frac{30}{180}$$

Step 3 Solve the proportion.

$$\begin{aligned}\frac{x}{100} &= \frac{30}{180} \\ x \cdot 180 &= 100 \cdot 30 \\ 180x &= 3,000 \\ x &= 16\frac{2}{3} \approx 16.7\%\end{aligned}$$

Solution The percent of the discount was about 16.7%.

Interest is the amount earned when money is deposited in a savings account or other type of investment. Interest can also be the extra amount paid when money is borrowed.

The original amount invested or borrowed is called the **principal**. To compute **simple interest**, solve a proportion to find the amount of interest earned in 1 year. Then, multiply by the number of years. Alternatively, you can use the simple interest formula.

Simple Interest Formula

$$I = prt$$

where I is the amount of the interest, p is the principal, r is the interest rate expressed as a decimal, and t is the time in years

Example 4

Dante invests \$3,500 and earns 2% in simple interest each year. Assuming he makes no additional deposits or withdrawals, how much will his investment be worth in 5 years?

Strategy Find the interest earned in 1 year. Multiply by the number of years. Then, add this to the principal. Check your answer using the simple interest formula.

Step 1

Write and solve a proportion to find the interest for 1 year.

$$\begin{aligned}\frac{2}{100} &= \frac{x}{3500} \\ 2 \cdot 3,500 &= 100 \cdot x \\ 7,000 &= 100x \\ 70 &= x\end{aligned}$$

In 1 year, the principal earns \$70 in interest.

Step 2

Multiply by the number of years.

$$5 \cdot 70 = 350$$

Step 3

Add the total interest to the principal.

$$350 + 3,500 = 3,850$$

Step 4

Check your answer using the simple interest formula.

$$2\% = \frac{2}{100} = 0.02, \text{ so:}$$

$$I = prt = 3,500 \cdot 0.02 \cdot 5 = 70 \cdot 5 = 350$$

This is the same amount of interest found using proportions, so the sum will be the same, 3,850.

Solution In 5 years, the investment will be worth \$3,850.

The **percent error** shows how close an estimate or measurement is to the actual value. Percent error is often used when describing a scientific experiment.

Example 5

John needs to measure his windows so he can buy new curtains. He measured the height of his windows to be $4\frac{3}{4}$ feet. The actual height is 5 feet. Find the percent error in John's measurement.

Strategy Find the absolute value of the difference between the estimated and actual heights. Use the actual height to write a proportion. Solve to find the percent.

Step 1

Find the positive difference between John's measurement and the actual measurement.

$$\left| \text{John's measurement} - \text{actual measurement} \right| = \left| 4\frac{3}{4} - 5 \right| = \left| -\frac{1}{4} \right| = \frac{1}{4}$$

The difference is $\frac{1}{4}$ foot.

Step 2

Set up a proportion.

You are looking for the percent error, so the first ratio should represent the percent.

The second ratio will compare the difference between the measurements to the actual measurement.

$$\frac{x}{100} = \frac{\frac{1}{4}}{5}$$

Step 3

Solve the proportion.

$$\begin{aligned} \frac{x}{100} &= \frac{\frac{1}{4}}{5} \\ x \cdot 5 &= 100 \cdot \frac{1}{4} \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

Solution The percent error in John's measurement was 5%.

2 COACHED EXAMPLE

The chart shows population changes in two towns over a period of 10 years. Which town had the greater percent increase in its population?

Population Changes in Ten Years

	2000	2010
Riverton	38,500	41,200
Mountainside	14,700	16,500

Determine the amount of the increase for Riverton.

$$41,200 - 38,500 = \underline{\hspace{2cm}}$$

Write a ratio for the percent increase:

$$\frac{\text{amount of increase}}{\text{original population}} = \frac{\boxed{\hspace{1cm}}}{38,500}$$

Set that ratio equal to $\frac{r}{100}$. Then solve for r , the percent increase for Riverton.

$$\frac{\boxed{\hspace{1cm}}}{38,500} = \frac{r}{100}$$

$$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}} \cdot r$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \approx r$$

So, the population of Riverton increased by approximately $\underline{\hspace{1cm}}\%$.

Determine the amount of the increase for Mountainside.

$$16,500 - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Write a ratio for the percent increase:

$$\frac{\text{amount of increase}}{\text{original population}} = \frac{\boxed{\hspace{1cm}}}{\boxed{\hspace{1cm}}}$$

Set that ratio equal to $\frac{m}{100}$. Then solve for m , the percent increase for Mountainside.

$$\frac{\boxed{\hspace{1cm}}}{\boxed{\hspace{1cm}}} = \frac{m}{100}$$

$$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}} \cdot m$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \approx m$$

The town of $\underline{\hspace{2cm}}$ had a greater percent increase in its population.

3 LESSON PRACTICE

- 1 The number of students enrolled at a community college increased by 12% from last year to this year. The enrollment last year was 15,500.

Use numbers from the box to complete each statement.

The amount of the increase in enrollment was _____.

The number of students enrolled this year is _____.

186
1,860
13,640
15,686
17,360

- 2 Select the items for which the discount is greater than 15%. Circle all that apply.

- A. \$29.95 box set of CDs on sale for \$19.95
- B. \$64.49 pair of shoes on sale for \$55
- C. \$85 radio on sale for \$74.49
- D. \$145 suitcase on sale for \$114.99

- 3 Henry works in a store where he earns a 3% commission on everything he sells.

Part A

What will be his total commission on a day when he sells three items, priced at \$86, \$245, and \$295?

Part B

How much must he sell to earn \$60 in commission? Show your work.

These charts shows state sales tax as of 2011. Use this chart for problems 4–5.

Alabama	4%
Florida	6%
Georgia	4%
Hawaii	4%
Idaho	6%
Indiana	7%
Iowa	6%
Kentucky	6%

Louisiana	4%
Maryland	6%
Michigan	6%
Mississippi	7%
New York	4%
North Dakota	5%
Oregon	0%
Pennsylvania	6%

Rhode Island	7%
South Carolina	6%
South Dakota	4%
Tennessee	7%
Vermont	6%
West Virginia	6%
Wisconsin	5%
Wyoming	4%

- 4** Students in various states are attending a marching band conference. Each uniform costs \$135 before state sales tax is added. How much will the uniforms cost, with tax, for each group of students?

10 students from Idaho _____

5 students from Mississippi _____

15 students from Hawaii _____

8 students from Wisconsin _____

- 5** A customer paid \$682.50, including sales tax, for a table priced at \$650. In which of these states could the customer live? Explain your answer.

6 Five people each invested \$1,500 in an account that earns simple interest. Assuming no other deposits or withdrawals were made, did each investment earn more than \$100? Select Yes or No.

- A.** 2 years at 4.2% Yes No
- B.** 1.3% for 6 years Yes No
- C.** 8 years at 0.75% Yes No
- D.** 5.2% for 1 year Yes No
- E.** 4 years at 2.8% Yes No

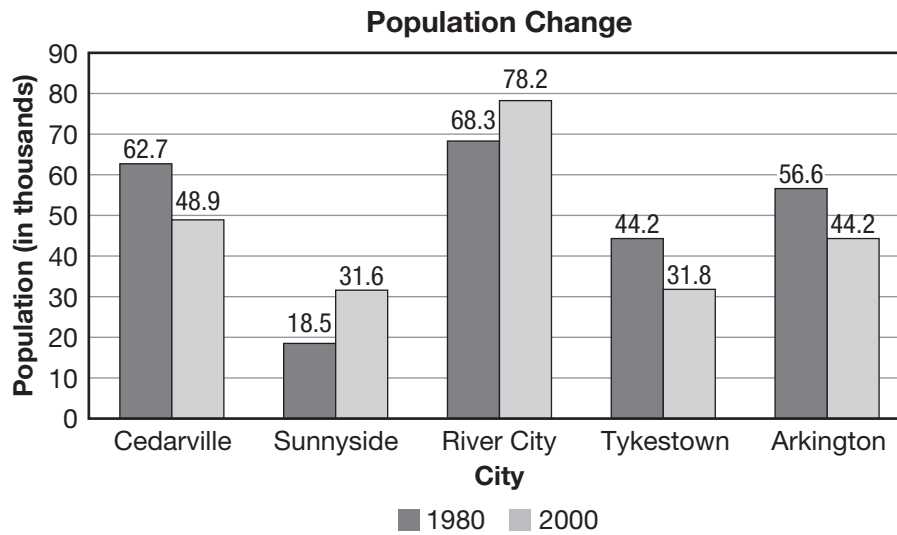
7 Julie estimated her town's population at 77,500. The actual population of her town is 75,000. Find the percent error.

percent error: _____

8 Neil will pay one-third of a restaurant bill that is \$72, including tax. He decides to tip 20% on the amount he pays for the bill. What is the total amount Neil will pay? Show your work.

9 Gayle can use her 12% employee discount once a week. Last Saturday, she bought a \$45 sweater at $\frac{1}{3}$ off and used her discount on a suit marked \$160. On which purchase did she save more money? How much more?

- 10 This graph shows population changes in several cities.



Which city or cities had a population decrease of over 25%? Justify your answer.