Mathematics

English Language Arts

common core Performance Coach

8

Sample Lesson

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CONTENTS

Writing Equivalent Numerical Expressions

Student Edition pages 30–41

LESSON OVERVIEW

Objectives

- Explore the properties of integer exponents
- Apply the properties of integer exponents to generate equivalent numerical expressions

Discussion Questions

MP3 How do you know when an expression containing powers is fully simplified?

MP7 Explain the relationship between the rules for multiplying and dividing powers.

Standard

8.EE.1

Key Terms

base
exponent
factor
negative exponent property
power
power of a power property
power of a product property
power of a quotient property
product of powers property
quotient of powers property
zero exponent property

Differentiation

LESSON SUPPORT

Have students solve problems in a two-column chart. Column one: they provide each step they take in solving the problem. Column two: they provide a brief explanation of each step. Model this activity for students.

LESSON EXTENSION

Have students explain which of the following are undefined: 1^0, 0^1, 0^{-1}, 1^1, 1^{-1}.

In each example, have students replace the base with a variable and then simplify. Ask: *How is simplifying an exponential expression with a variable base different from simplifying an exponential expression with a numerical base?*

1 GETTING THE IDEA

Lesson Opener

Present students with 4 + 4 + 4 + 4. Ask: *How can you write an equivalent expression using multiplication?* Then present students with a repeated multiplication problem such as 4 × 4 × 4 × 4. Ask: *How can we rewrite and simplify repeated multiplication problems?* This will help connect exponents to something they already know and review the fact that an exponent is just shorthand for repeated multiplication.

▲ ***ELL Support*** *The words* base, power, *and* factor *all have mathematical and nonmathematical definitions. Discuss the various definitions as a class. Have students add each word to their dictionary. Students should include two to three full sentences for each word.*

Examples 1 and 2

When addressing the product of powers property, have students write out each term in the expression and count the number of times the base is multiplied by itself. For example,

$$7^3 \times 7^5 = (7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7)$$
$$= (7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7)$$

When addressing the quotient of powers property, have students write out each term in the expression and begin simplifying by crossing out the common factors in the numerator and denominator.

▲ ***Common Errors*** *Errors occur when students forget that the properties of exponents only work when the bases are the same. Provide students with problems to solve that include some with common bases and some with different bases. Discuss as a class.*

Examples 3, 4, and 5

These examples require students to apply the power of a power, power of a product, and power of a quotient properties. With each problem, encourage students to write out each term in the expression. Review the distributive property when addressing the power of a product and power of a quotient properties.

Examples 6 and 7

Assist students in understanding why the zero and negative exponent properties work. Present them with a two-column table. The first column consists of seven rows listing 2^3 to 2^{-3}. The second column provides the answers to the powers for 2^3 to 2^1, leaving the last four rows blank. Ask: *What patterns do you see*? *What comes next*? Discuss that as the exponent decreases by 1, the values of the powers are halved.

▲ ***Common Errors*** *Emphasize the importance of writing mathematical expressions clearly. For example, 4^{-5} can be confused with $4 - 5$.*

Example 8

This example combines several of the exponential properties. Remind students that exponents are shorthand for repeated multiplication and that multiplication is commutative; therefore, there are several ways to solve the problem. They can start with any of the properties.

▲ ***Journal Prompt*** **MP3** *How many different ways can you find to solve this problem? Show your work for each approach.*

DOMAIN 2

2 COACHED EXAMPLE

Monitor students as they work through the Coached Example. As needed, assist them in naming the different properties they are using. Students may know how to use the properties but not know their exact names. Guide them through each step, reminding them that exponents are repeated multiplication and that they can write out each term of the expression to help them determine which property to use.

For answers, see page A4.

3 LESSON PRACTICE

As students are working, pay special attention to problems 6, 7 and 9. For problem 7, there may be more than one way to represent 81. For problems 6 and 9, you may want to solve similar problems as a class before students try them on their own.

For answers, see page A5.

8.EE.1

Writing Equivalent Numerical Expressions

1 GETTING THE IDEA

You can use properties of exponents to help simplify expressions containing powers. Remember, an **exponent** tells how many times to use a number, called the **base**, as a **factor**. In an exponential term, the exponent is sometimes referred to as **power**.

base — exponent or power

$$\underbrace{3^4}_{\text{term}} = \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors of 3}} = 81$$

Product of Powers Property

To multiply exponential terms with the same base, add the exponents.
$x^a \cdot x^b = x^{(a+b)}$ where x is a real number and a and b are integers.

For example, $4^2 \cdot 4^3 = 4^{(2+3)} = 4^5$

because $4^2 \cdot 4^3 = (4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$

$= 16 \cdot 64 = 1{,}024$

Quotient of Powers Property

To divide exponential terms with the same base, subtract the exponents.
$\frac{x^b}{x^a} = x^{(b-a)}$, where x is a real number and a and b are integers.

For example, $\frac{5^5}{5^2} = 5^{5-2} = 5^3$

because $\frac{5^5}{5^2} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$

$= \frac{3{,}125}{25} = 125$

Example 1

Write $7^3 \cdot 7^5$ using a single exponent.

Strategy **Use the properties of exponents to multiply.**

Step 1 Use the product of powers property.

The terms have the same base, so add the exponents.

$7^3 \cdot 7^5 = 7^{(3 + 5)}$

Step 2 Simplify.

$= 7^{(3 + 5)}$

$= 7^8$

Solution $7^3 \cdot 7^5 = 7^8$

Example 2

Write $\frac{9^6}{9^4}$ using a single exponent.

Strategy **Use the properties of exponents to divide.**

Step 1 Use the quotient of powers property.

The terms have the same base, so subtract the exponents.

$\frac{9^6}{9^4} = 9^{(6 - 4)}$

Step 2 Simplify.

$= 9^{(6 - 4)}$

$= 9^2$

Solution $\frac{9^6}{9^4} = 9^2$

Power of a Power Property

To raise an exponential term to a power, multiply the exponents.

$(x^a)^b = x^{ab}$, where x is a real number and a and b are integers.

For example, $(7^3)^2 = 7^{3 \cdot 2} = 7^6$

because $(7^3)^2 = (7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)$

$= 343 \cdot 343 = 117{,}649$

Power of a Product Property

To raise a product to a power, raise each factor to the same power.

$(xy)^a = x^a y^a$, where x and y are real numbers and a is an integer.

For example, $(3 \cdot 2)^4 = 3^4 \cdot 2^4$

because $(3 \cdot 2)^4 = (3 \cdot 2) \cdot (3 \cdot 2) \cdot (3 \cdot 2) \cdot (3 \cdot 2)$

$= (3 \cdot 3 \cdot 3 \cdot 3) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$

$= 81 \cdot 16 = 1{,}296$

Power of a Quotient Property

To divide exponential terms with the same base, subtract the exponents.

$\frac{x^b}{x^a} = x^{(b-a)}$, where x is a real number and a and b are integers.

For example, $\frac{5^5}{5^2} = 5^{5-2} = 5^3$

because $\frac{5^5}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$

$= \frac{3{,}125}{25} = 125$

Example 3

Write $(8^3)^5$ using a single exponent.

Strategy **Use the properties of exponents to raise an exponential term to a power.**

Step 1 Use the power of a power property.

Multiply the exponents.

$(8^3)^5 = 8^{3 \cdot 5}$

Step 2 Simplify.

$= 8^{3 \cdot 5}$

$= 8^{15}$

Solution $(8^3)^5 = 8^{15}$

Example 4

Evaluate: $(4 \cdot 2)^2$

Strategy **Use the properties of exponents.**

Step 1 Use the power of a product property.

Raise each factor to the power of 2.

$(4 \cdot 2)^2 = 4^2 \cdot 2^2$

Step 2 Evaluate each exponential term.

$= 4^2 \cdot 2^2$

$= (4 \cdot 4) \cdot (2 \cdot 2)$

$= 16 \cdot 4$

Step 3 Multiply.

$= 16 \cdot 4$

$= 64$

Solution $(4 \cdot 2)^2 = 64$

Example 5

Evaluate: $\left(\frac{3}{8}\right)^4$

Strategy **Use the properties of exponents.**

Step 1 Use the power of a quotient property.

Raise the numerator and the denominator to the power of 4.

$$\left(\frac{3}{8}\right)^4 = \frac{3^4}{8^4}$$

Step 2 Evaluate the exponential terms in the numerator and the denominator.

$$= \frac{3^4}{8^4}$$

$$= \frac{3 \cdot 3 \cdot 3 \cdot 3}{8 \cdot 8 \cdot 8 \cdot 8}$$

$$= \frac{81}{4{,}096}$$

Solution $\left(\frac{3}{8}\right)^4 = \frac{81}{4{,}096}$

An exponential term may include an exponent of 0 or a negative exponent.

Zero Exponent Property

Any nonzero number raised to the power of 0 is 1.

$x^0 = 1$, where $x \neq 0$.

For example: $125^0 = 1$

Negative Exponent Property

For any nonzero number x and integer a,

$x^{-a} = \frac{1}{x^a}$.

For example, $5^{-4} = \frac{1}{5^4}$

because $5^{-4} = \frac{1}{5^4} = \frac{1}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{625}$

Example 6

Evaluate: $(-4)^0$

Strategy **Use the properties of exponents.**

Use the zero exponent property to simplify the expression.

Any nonzero number raised to the 0 power is equal to 1.

$(-4)^0 = 1$

Solution $(-4)^0 = 1$

Example 7

Evaluate: 4^{-3}

Strategy **Use the properties of exponents.**

Step 1 Use the negative exponent property.

Take the reciprocal of the base and change the sign of the exponent.

$4^{-3} = \frac{1}{4^3}$

Step 2 Evaluate the exponential term in the denominator.

$= \frac{1}{4^3}$

$= \frac{1}{4 \cdot 4 \cdot 4}$

$= \frac{1}{64}$

Solution $4^{-3} = \frac{1}{64}$

Example 8

Simplify: $(3^2)^3 \cdot (5^0 \cdot 3^{-1})$

Strategy **Use the properties of exponents to simplify the expression.**

Step 1 Use the power of a power property to simplify $(3^2)^3$.

$$(3^2)^3 \cdot (5^0 \cdot 3^{-1}) = 3^6 \cdot (5^0 \cdot 3^{-1})$$

Step 2 Use the zero exponent property to simplify 5^0.

$$= 3^6 \cdot (5^0 \cdot 3^{-1})$$
$$= 3^6 \cdot (1 \cdot 3^{-1})$$

Step 3 Use the identity property of multiplication to simplify $1 \cdot 3^{-1}$.

$$= 3^6 \cdot (1 \cdot 3^{-1})$$
$$= 3^6 \cdot 3^{-1}$$

Step 4 Use the product of powers property to simplify $3^6 \cdot 3^{-1}$.

$$= 3^6 \cdot 3^{-1}$$
$$= 3^{6 + (-1)}$$
$$= 3^5$$

Solution $(3^2)^3 \cdot (5^0 \cdot 3^{-1}) = 3^5$

2 COACHED EXAMPLE

Simplify: $(4^3)^4 \cdot (4^{-2} \cdot 3^0)$

First, simplify $(4^3)^4$.

Use the power of a ________________ property.

To raise an exponential term to a power, ________________ the exponents.

$(4^3)^4 \cdot (4^{-2} \cdot 3^0) = 4^{\square} \cdot (4^{-2} \cdot 3^0)$

Next, simplify 3^0.

Use the ________________ exponent property.

$= 4^{12} \cdot (4^{-2} \cdot 3^0)$

$= 4^{12} \cdot (4^{-2} \cdot \_\_\_\_\_\_\_\_)$

Then, simplify $4^{-2} \cdot 1$.

$= 4^{12} \cdot (4^{-2} \cdot 1)$

$= 4^{12} \cdot \_\_\_\_\_\_\_\_$

Last, simplify $4^{12} \cdot 4^{-2}$.

Use the product of ________________ property.

$= 4^{12} \cdot 4^{-2}$

$= 4^{\square + \square}$

$= 4^{\square}$

$(4^3)^4 \cdot (4^{-2} \cdot 3^0) =$ ________

1 Select True or False for each equation.

A. $9^3 \cdot 9^4 = 9^{12}$ ○ True ○ False

B. $(5^2)^5 = 5^{10}$ ○ True ○ False

C. $(2 \cdot 4)^6 = 2^6 \cdot 4^6$ ○ True ○ False

D. $8^0 = 0$ ○ True ○ False

E. $\frac{7^8}{7^4} = 7^2$ ○ True ○ False

F. $3^{-2} = \frac{1}{9}$ ○ True ○ False

2 Is each expression equivalent to 16? Select Yes or No.

A. $\frac{4^8}{4^6}$ ○ Yes ○ No

B. 16^0 ○ Yes ○ No

C. $2^1 \cdot 2^3$ ○ Yes ○ No

D. $(2 \cdot 2)^2$ ○ Yes ○ No

E. 8^2 ○ Yes ○ No

F. 4^{-2} ○ Yes ○ No

3 Circle the exponential expression or value that makes each equation true.

$5^2 \cdot \begin{array}{|c|} \hline 5^3 \\ \hline 5^4 \\ \hline 5^{12} \\ \hline \end{array} = 5^6$

$2^0 = \begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline 2 \\ \hline \end{array}$

4 Kate ate $\frac{1}{8}$ of a veggie pizza. Which expressions are equivalent to $\frac{1}{8}$? Circle all that apply.

A. 2^{-3}

B. $(-8)^1$

C. $\left(\frac{32}{4}\right)^{-1}$

D. $8^8 - 8^9$

E. $\frac{8^8}{8^9}$

F. $\left(\frac{1}{8}\right)^0$

G. $(2^3)^{-1}$

5 Compare the value of each expression to 64. Write the expression in the correct box.

$2^3 \cdot 2^2$	$\frac{4^7}{4^3}$	$(8^2)^2$	64^0	$(2 \cdot 4)^3$

Less Than 64	Greater Than 64

6 Draw a line from each expression to its equivalent expression.

A. $(9^3)^6$	9^3
B. $(9 \cdot 3)^6$	$\frac{9^6}{3^6}$
C. $\left(\frac{9}{3}\right)^6$	$9^6 \cdot 3^6$
D. $\frac{9^6}{9^3}$	9^{18}

7 Luis used the properties of exponents to write two equations equal to 81. Use exponential terms from the box to complete the equations.

$81 = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$

$81 = \_\_\_\_\_\_ \div \_\_\_\_\_\_$

3^1
3^3
3^4
9^4
9^6
9^8
$(3^2)^3$

8 For each expression in the table, indicate with an "X" whether the value of the expression is less than 1, equal to 1, or greater than 1.

Expression	Less Than 1	Equal to 1	Greater Than 1
$(4 \cdot 7)^{-2}$			
$\left(\frac{18}{6}\right)^3$			
$(12^0)^1$			

9

Part A

Write 5^8 as a quotient of two exponential terms with the same base in four different ways. Use only positive exponents.

Part B

Write 5^8 as a quotient of two exponential terms with the same base in four different ways. Use negative or zero exponents.

Part C

How many ways can you write 5^8 as a quotient of two exponential terms? Explain your reasoning.